Frequency and angular bandwidth of acousto-optic deflectors with ultrasonic walk-off

Jean-Claude Kastelik\textsuperscript{a,b,c,*}, Samuel Dupont\textsuperscript{a,b,c}, Konstantin B. Yushkov\textsuperscript{d}, Joseph Gazelet\textsuperscript{a,b,c}

\textsuperscript{a}Université Lille Nord de France, 59000 Lille, France
\textsuperscript{b}Université de Valenciennes et du Hainaut-Cambrésis, IEMN-DOAE, Le Mont Houy, 59313 Valenciennes Cedex 9, France
\textsuperscript{c}CNRS, UMR 8520, 59650 Villeneuve d’Ascq, France
\textsuperscript{d}National University of Science and Technology “MISIS”, Acousto-Optical Research Center, 4 Leninsky Prospect, 119049 Moscow, Russia

\begin{abstract}
In the paper, bandwidth parameters of acousto-optical deflectors (AODs) are analyzed from the point of view of acoustical anisotropy. Equations for bandwidth and central frequency of AOD are derived for arbitrary propagation direction of ultrasound in optically uniaxial crystals. The phenomenon of bandwidth shift due to phase mismatch at the central frequency is studied theoretically and verified experimentally.
\end{abstract}

1. Introduction

Acousto-optical deflectors (AODs) is a class of all-solid-state photonic devices that are used for continuous scanning of laser light beams\cite{1}. The deflector is characterized by a wide bandwidth of diffraction, and deflection angle of light is proportional to the frequency of driving ultrasound. Broadband ultrasonic deflection of light is applied in RF spectrum analyzers\cite{2}, multichannel WDM commutators\cite{3–5}, optical tweezers for particle trapping\cite{6–8}, image scanners\cite{9–11}, fringe projectors\cite{12–14}, and frequency shifters\cite{15,16}. Flat frequency response and precise knowledge of AOD bandwidth is required for most applications.

Anisotropic acousto-optical diffraction in crystals was proposed for deflection of laser beams independently by Dixon\cite{17} and by Lean et al.\cite{18}. Later, Warner et al. introduced axial AOD in paratellurite that used optical activity of the crystal\cite{19}. An important feature of that device was midband degeneracy caused by double Bragg scattering. Application of that effect for increasing bandwidth of deflection was demonstrated by Voloshinov et al.\cite{20} and by Chang and Hecht\cite{21}. Meanwhile, Yano et al. developed an off-axial AOD that was free of midband degeneracy\cite{22}. That device featured strong acoustical energy walk-off that is due to extremely high elastic anisotropy of slow shear acoustic wave in paratellurite\cite{23}. The walk-off angle in (1\text{\text{10}}) plane of paratellurite quickly grows when the acoustical wave vector is tilted from \text{\text{110}}/38-axis and reaches the maximum of 54°\cite{24–26}. Theoretical and experimental analysis of two-coordinate AOD was performed by Maák et al.\cite{27}. Several recent works were also devoted to the problem of thermal impact on performance of AOD\cite{28,29}.

Bragg phase matching in AOD is realized in tangential geometry, and phase mismatch magnitude is approximately proportional to the square of ultrasonic frequency deviation, providing zero derivative at the central frequency\cite{20}. The tangential phase matching can be obtained only when the slow optical eigenwave is incident, and the fast eigenwave is diffracted\cite{1}.

Extensive development of laser technologies establishes a challenge for searching new materials that can be applied in acousto-optics. Crystals with stronger optical anisotropy, including biaxial ones, are being studied. The aim of this work is to generalize methods for calculation of basic parameters of AOD in crystals with optical and acoustical anisotropy. Special attention is paid to the shift of AOD bandwidth when the device operates under phase mismatch condition relatively to the central frequency of tangential phase matching. This phenomenon was observed for AOD on the base of paratellurite (TeO\textsubscript{2}) crystals.
2. Theoretical analysis

2.1. Coupling of modes: plane waves approximation

We consider plane wave approximation of anisotropic Bragg diffraction in a crystal with strong elastic anisotropy. The piezotransducer with the length \( l \) along \( x \)-axis launches in the crystal an ultrasonic wave that has a walk-off angle of energy flow \( \psi \), i.e. the angle between the phase velocity \( \mathbf{V} \) and the group velocity \( \mathbf{U} \). As a result, the cross-section of acoustical beam in the crystal along \( \xi \)-axis has a length \( L = l \cos \psi \) that is referred as the length of acousto-optical interaction. Schematic design of AOD is shown in Fig. 1. Typical regime of deflector operation is characterized by approximate phase matching, when a finite mismatch \( H \neq 0 \) is present. Diffraction efficiency in Bragg regime can be described in terms of coupled modes theory. The transmission coefficient \( T \) is defined as ratio of diffracted light intensity to the incident light intensity. The following transmission function can be obtained for Bragg diffraction [30]:

\[
T(P, h) \propto \frac{\sin^2 \left( \frac{\pi}{2} \sqrt{P/P_0 + (hl/\pi)^2} \right)}{P/P_0 + (hl/\pi)^2},
\]

where \( h = H \cos \psi \) is the \( x \)-component of the mismatch, \( P \) is the ultrasonic power, and \( P_0 \) is the characteristic ultrasonic power for 100% diffraction efficiency. Eq. (1) determines admissible mismatch magnitudes \(|h| = \phi_\text{max} \) for different relative levels of driving ultrasonic power \((P/P_0)\) and different criteria of bandwidth ripples. For a conventional \(-3 \, \text{dB} \) criterion at \( P/P_0 = 1 \), the maximum normalized mismatch is \( \phi_\text{max}/\pi = 0.8 \). For some applications, better uniformity of transmission function is required. For example, 10% of intensity nonuniformity that corresponds to \(-0.5 \, \text{dB} \) reduction of diffraction efficiency restricts maximum mismatch as \( \phi_\text{max}/\pi = 0.34 \).

2.2. Wave vector diagrams for phase mismatch

In the following analysis we use the method of wave vector diagrams. The wave vector of incident light \( \mathbf{k}_i \) is coupled with the wave vector of diffracted light \( \mathbf{k}_d \) via the wave vector of ultrasound \( \mathbf{K} \) and the phase mismatch vector \( \mathbf{H} \):

\[
\mathbf{k}_d = \mathbf{K} + \mathbf{H} = \mathbf{k}_d.
\]

It is important to note that coupled modes theory of acoustooptic diffraction justifies that method in the plane-waves approximation [31–33]. The phase mismatch magnitude is determined as projection of wave vectors difference to the \( \xi \)-axis, i.e. orthogonally to acoustical energy flow [34,35]:

\[
H = k_{d,\xi} - k_{i,\xi} - K_i.
\]

Thus, the phase mismatch vector is orthogonal to the acoustical group velocity vector \( \mathbf{U} \). In the chosen axes \( \{x,y\} \), the components of mismatch vector are equal to \( \mathbf{H} = \{-h, -h \tan \psi\} \) (according to the accepted choice of signs, the magnitude of \( \psi \) hereinafter is negative).

For further analysis, we use a general representation of optical normal surface sections as two ellipses. This approach covers most practical cases of crystals with optical anisotropy that are recently used in acousto-optics: uniaxial crystals with arbitrary orientation of ultrasonic wave vector, and also biaxial crystals in (001) and (100) planes. However, we will use notation that is typical for a positive uniaxial crystal, i.e. \( n_o \) and \( n_e \) for refractive indices of ordinary and extraordinary waves that propagate orthogonally to the optical axis; \( n_o \) and \( n_e \) are used for the fast and slow waves correspondingly that propagate in the principal optical plane of the crystal. Wave vector diagram of Bragg phase matching with both high-frequency and low-frequency diffraction in AOD is shown in Fig. 2. Tilt angle of the acoustical wave vector \( \mathbf{K} \) relatively \([110]\)-axis is designated as \( \alpha \).

Consider the case when exact phase matching \((H = 0)\) takes place. For the low-frequency branch of diffraction \( \mathbf{K}_e = \mathbf{K}_i \) and \( \mathbf{k}_i = \mathbf{k}_i \), while for the high-frequency branch \( \mathbf{K}_e = \mathbf{K}_o \) and \( \mathbf{k}_o = \mathbf{k}_o \) correspondingly. Ultrasonic frequency \( f = |\mathbf{K}_e|V/(2\pi) \) is the central frequency of AOD bandwidth. Correspondent wave vector of diffracted light equals to \( \mathbf{k}_d = \mathbf{k}_i + \mathbf{K}_e \). If the frequency of ultrasound is varied, i.e. \( K = K_o + \Delta K \), the phase mismatch appears:

\[
\mathbf{k}_c + \Delta \mathbf{K} + \mathbf{H} = \mathbf{k}_d.
\]

The wave vector of diffracted light \( \mathbf{k}_d \) corresponds to the deflected light beam.

Under perfect phase matching, the diffracted wave vector \( k_c = \{x, y\} \) is determined by the following components:

\[
x_c = \frac{\omega}{c} \sqrt{n_o^2 \cos^2 \alpha + n_e^2 \sin^2 \alpha},
\]

\[
y_c = \frac{\omega}{c} \sqrt{n_o^2 \cos^2 \alpha + n_e^2 \sin^2 \alpha}.
\]

The tilt angle of diffracted wave vector relatively to \( x \)-axis equals to

Fig. 1. Schematic layout representing off-axial AOD unit.

Fig. 2. Wave vector diagram of tangential phase matching.
\[ \theta_c = -\arctan \left( \frac{n_y^2 - n_e^2}{n_{6}^2 \cos^2 \alpha + n_{5}^2 \sin^2 \alpha} \right) \]  

Hereinafter, the common relation, \( \omega/\epsilon = 2\pi/\lambda \), is used for optical frequency \( \omega \) and wavelength in the vacuum \( \lambda \).

2.3. Deflection bandwidth and scanning angle

When the frequency of ultrasound is different from central frequency \( f_0 \), the diffracted wave vector \( \mathbf{k}_d \) is scanning from \( \mathbf{k}_{0_d} \) to \( \mathbf{k}_{h_d} \). Since the boundaries of AOD bandwidth are defined by substitution: \( x_0 = x_c - h \), correspondent diffracted wave vectors can be found based on normal surface equation for the fast optical eigen-wave in the crystal. Thus, deviations of acoustical wave number are calculated from reduced wave vector diagrams that are shown in Fig. 3:

\[ \Delta K = \frac{h(n_{6}^2 - n_{5}^2) \cos \alpha \sin \alpha \pm n_{10} n_{1} \sqrt{h(2x_c - h)}}{(c/\omega)^2 x_{6}^2} + h \tan \psi. \]  

From this equation, one can see that optical anisotropy \( (n_{9} \neq n_{1}) \) and acoustical walk-off angle \( \psi \) result in a certain asymmetry of AOD bandwidth, i.e. \( |\Delta K| \neq |\Delta K_\omega| \). On the other hand, the whole bandwidth of the AOD is not influenced by acoustical walk-off:

\[ \Delta K = \Delta K_\omega - \Delta K_\mu = 2 \frac{n_{10} n_{1} \sqrt{h(2x_c - h)}}{(c/\omega)^2 x_{6}^2}. \]  

Another quantity that is closely related to the bandwidth of AOD is the maximum deflection angle. The following equation results from the wave vector diagram in Eq. (4):

\[ \tan(\theta_c + \Delta \theta_x) = \tan \theta_c \pm \frac{D}{1 + \tan^2 \theta_c \pm D \tan \theta_c}. \]  

Thus, the whole scanning range of diffracted beam \( \Delta \theta = \Delta \theta_x - \Delta \theta_y \) may be found as

\[ \tan \Delta \theta = \frac{2D}{1 + \tan^2 \theta_c - D^2}. \]  

From Eqs. (11) and (12), it is obvious that neither \( \Delta \theta_x \) nor \( \Delta \theta_y \) values depend on the acoustical walk-off angle \( \psi \), however both of them are influenced by optical anisotropy, because the values of \( \theta_c \) and \( x_c \) both depend on the value of \( (n_{6}^2 - n_{5}^2) \).

2.4. AOD with midband mismatch

In a general case, the AOD can be detuned from exact phase matching at the central frequency. Thus, a midband mismatch appears, and perfect phase matching is obtained at two separate ultrasonic frequencies [1]. Conventionally, the boundary mismatch \( H_a \) is taken equal to the central mismatch \( H_c \) that provides maximum deflection bandwidth with the given criterion for ripples magnitude. Nevertheless, for some applications it can be advisable to choose \( H_c < H_a \), that provides greater uniformity of diffraction efficiency in the center of transmission bandwidth. As follows from geometrical constructions, the bandwidth \( \Delta K \) and the scanning range \( \Delta \theta \) of AOD in this case are still determined by Eqs. (9) and (12), but the mismatch parameter equals to

\[ h = h_c + h - (H_a + H_c) \cos \psi. \]  

Thus, the bandwidth and the scanning range of AOD can be increased. E.g., if \( h_c = h_0 \), the bandwidth increase approximately by a factor of \( \sqrt{2} \) takes place compared to \( h_c = 0 \).

In practice, misalignment of the AOD is performed by varying the Bragg incidence angle and the central frequency of ultrasound. Correspondent ultrasonic wave number is calculated by substituting \( x = x_c - h_c \) into the equation for optical normal surface of the slow eigenwave:

\[ K = y_c - h_c \tan \psi + \frac{(x_c - h_c)(n_{6}^2 - n_{5}^2) \cos \alpha \sin \alpha}{(c/\omega)^2 x_{6}^2} \pm \frac{n_{10} n_{1} \sqrt{x_{6}^2 - (x_c - h_c)^2}}{(c/\omega)^2 x_{6}^2}, \]  

where \( x_m = (\omega/c)(n_{6}^2 \cos^2 \alpha + n_{5}^2 \sin^2 \alpha)^{1/2} \). Bragg incidence angle follows the equation:

\[ \tan \theta = \frac{K_0 - y_c + h_c \tan \psi}{x_c - h_c}. \]  

Eqs. (14) and (15) are valid for calculating parameters of both low-frequency and high-frequency diffraction in AOD: minus sign is taken in Eq. (14) for the case \( K = K_f \); vice versa, for \( K = K_0 \), plus sign is used.

It should be noted, that the deflection angle \( \Delta \theta \) and the incidence Bragg angle \( \theta \) in Eqs. (7) and (15) are calculated for optical waves inside the crystal. After the refraction of light by the output optical facet of AOD, the scanning angle will be increased approximately by a factor of refractive index of diffracted light:

\[ \Delta \theta' = \arcsin \frac{n_{10} n_{1} \sin \Delta \theta}{\sqrt{n_{6}^2 \cos^2 (\alpha - \theta_c) + n_{5}^2 \sin^2 (\alpha - \theta_c)}}. \]  

For a positive crystal, \( n_0 = n_f \) and it can be concluded that \( \Delta \theta' \approx n_0 n_\theta \Delta \theta \).

3. Calculations and measurements

Analysis of Eq. (14) reveals that the central frequency of AOD is varied, if the deflection bandwidth is changed. Calculations show that the shift of central frequency is proportional to the magnitude of maximum phase mismatch \( \phi_{\text{max}} \). Thus, the frequency shift is higher for greater magnitudes of bandwidth ripples. Both ultrasonic walk-off and asymmetry of optical normal surfaces contribute to the frequency shift. However, mainly the shift for the
central frequency is determined by the variation of Bragg phase matching condition.

Central frequency and diffraction bandwidth are plotted versus magnitude of bandwidth ripples in Fig. 4 for different tilt angles of slow shear acoustical wave in (1T0) plane in paratellurite. Calculations were made for the length of the piezotransducer \( l = 1 \text{ mm} \). Both frequency shift and bandwidth of diffraction decrease with the piezotransducer length as \( l^{-1/2} \). Present calculations show that for smaller tilt angles of ultrasonic wave vector \( \alpha \), the shift of the central frequency is higher. Meanwhile, the bandwidth of diffraction changes not more than to 6% with the increase of tilt angle from \( \alpha = 6^\circ \) to \( 9^\circ \). Thus, the higher the tilt angle of ultrasound in the crystal, the less significant the shift of central frequency.

In principle, the length of the piezotransducer could be reduced to provide wider bandwidth of diffraction and to avoid bandwidth ripples, but a lack of diffraction efficiency is a substantial drawback of this approach [1]. Decrease of the piezotransducer length for the purpose of bandwidth flattening is especially undesirable in multifrequency applications of AODs. The lower the selectivity of Bragg diffraction, the higher are the phase grating intermodulation products that are caused by multiple rediffraction [13,36]. Thus, the adjustment of AOD with central mismatch and bandwidth ripples is preferable. Broadband electrical matching of piezotransducer with the RF driver is another important factor that determines frequency bandwidth and ripples depth of AODs. Efficiency of the electrical to ultrasonic power conversion by the piezotransducer decreases when detuning from its resonance frequency.

Experiments were performed with a high-frequency AOD in paratellurite with the tilt angle \( \alpha = 6^\circ \) and length of the piezotransducer \( l = 1.7 \text{ mm} \). This configuration of AOD has the phase velocity of ultrasound \( V = 653 \text{ m/s} \) and walk-off angle \( \psi = -44.6^\circ \). The bandwidth and central frequency calculated from Eqs. (9) and (12) are equal to \( \Delta f = 60 \text{ MHz} \) and \( f_c = K_h V/(2\pi) = 168.24 \text{ MHz} \) at the optical wavelength \( \lambda = 514 \text{ nm} \).

The measurements were made with a sweep frequency RF generator. Measurement results are plotted in Fig. 5 for different ripples depth criteria: 0 dB, −0.5 dB, and −3.0 dB. Without central dip, the bandwidth of \( \Delta f \approx 58.8 \text{ MHz} \) centered at \( f = 168.75 \text{ MHz} \) was obtained with −3 dB criterion (curve 1 in Fig. 5). Maximum diffraction efficiency over 80% was observed at \( P_d \approx 200 \text{ mW driving power} \). The disturbances of bandwidth shape resulted from imperfection of broadband electrical matching: measurements of RF reflection spectrum of electrical signal gave inhomogeneity of approximately 5 dB in the frequency range from 140 to 200 MHz.

Calculated shift of the central frequency for −3 dB depth of ripples (\( \phi_{\text{ms}}/\pi = 0.8 \)) was equal to 4.2 MHz for \( l = 1.7 \text{ mm} \). In the experiment, the −3 dB bandwidth with correspondent central dip was equal to \( \Delta f \approx 83.5 \text{ MHz} \) centered at 173.25 MHz (curve 3 in Fig. 5). Thus, the frequency shift of 4.5 MHz was experimentally observed.

### 4. Discussion

Eqs. (8), (9), (12), (14), and (15) are enough to calculate all main parameters of AOD in anisotropic media, namely, the bandwidth and the central frequency, also the scanning angle and the Bragg angle of light incidence. Analysis in Sections 2.2–2.4 covers a general case of a birefringent uniaxial crystal with acoustical walk-off and arbitrary value of midband mismatch. Nevertheless, for practical applications it is useful to give some remarks on the particular cases of the derived above formulæ.

#### 4.1. Positive and negative uniaxial crystals

One of the most important cases in practice is AOD in paratellurite that is a positive uniaxial crystal with rotatory power. For the diffraction in (1T0) plane, we should take \( n_1 = n_2 = n_o \). It is important to remember that rotatory power of paratellurite can seriously influence the parameters of AOD, if incident or diffracted optical waves propagate closely to the optical axis of the crystal. In this case, Eq. (14) can not be used, because normal surface sections of a crystal with optical activity are not approximated by ellipses [30]. Instead of this, a correction to the refractive indices is necessary: the Fresnel equation with the gyration tensor component \( C_{33} \) gives the solution \( n^2 = n_1^2 \pm (\sqrt{\Delta n^2 + C_{33}^2} - \delta_h) \), where \( n_1 \) and \( n_2 \) are the refractive indices calculated without rotatory power, and \( \delta_h = (n_1^2 - n_2^2)/2 \). Numerical analysis shows, that for tilt angles \( \alpha \geq 6^\circ \) rotatory power influence on parameters of paratellurite AOD is not crucial, however precision of frequency calculation with approximate equations can lay within several megahertz. It is also worth noting that approximation of normal surface sections as ellipses is not satisfactory for numerical evaluation of AOD parameters at \( \alpha = 0 \). For the positive crystal without rotatory power, \( n_o = n_i \), and from Eq. (7) it follows that \( \delta_h = 0 \).

Another practical case that can be derived from general equations corresponds to negative uniaxial crystals. Such materials that
are applied in acousto-optics are, e.g., KDP and lithium niobate. For this case, mutual permutation \( n_p \leftrightarrow n_g \) must be made in the formulae to correspond to traditional notations, as well as further substitution \( n_g = n = n_p \). Unlike the positive crystals, in negative crystals the section of optical normal for the fast eigenwave is an ellipse but not a circle, hence \( \theta_c \neq 0 \).

4.2. Estimations of anisotropy influence

In Eqs. (8), (9), and (12), optical anisotropy and acoustical walk-off is included together with the terms that determine phase mismatch. For better understanding of relative weight of these quantities, it is prominent to make some numerical estimations.

Let us examine Eq. (8) in detail. The maximum magnitude of phase mismatch \( h \) that corresponds to the boundaries of AOD bandwidth is inversely proportional to the dimension of the piezotransducer \( l \). For a typical AOD, the dimension of the transducer equals to several millimeters, hence typical magnitudes of mismatch are \( h = \phi_{\text{max}}/l \approx 10 \text{ cm}^{-1} \). On the other hand, the magnitude of \( x_c \) is inversely proportional to the optical wavelength \( \lambda \). Thus, the magnitude is of the order of \( x_c \approx 10^{-1} \). Therefore the approximation \( x_c - h = x_c \) gives an error of the order of only \( 0.01% \).

Influence of optical anisotropy on the symmetry of AOD bandwidth takes place only in negative crystals, where a non-zero term \((n_2^c - n_2^e)\) is present. However, the magnitude of that term is of the order of 0.1 (for example, in calcite it reaches 0.17, and in other negative crystals that are common in acousto-optics, birefringence is even smaller). Similar estimations can be made to describe influence of optical activity on the bandwidth shift of the AOD. For example, in paratellurite that has high rotatory power of 160(GL/mm at \( \lambda = 514 \text{ nm} \)), the gyration is of the order of \( \phi_{\text{gyr}} \approx 10^{-2} \). Thus, the bandwidth shift due to optical activity is also several orders smaller than that caused by phase mismatch.

Consider the last term in Eq. (8), \( \tan \psi \), describing acoustical walk-off. The highest known walk-off angle of bulk waves known in acoustics, \( \psi = 74^\circ \) is observed in (001) plane of paratellurite, and in (100) plane it is not higher than \( 54^\circ \) [24-26]. Thus, for all materials we can estimate \( \tan \psi \approx 1 \) or less.

Finally, we can conclude that both optical and acoustical anisotropy of crystals changes the symmetry of AOD bandwidth to the magnitude of the order of 1% or less. Thus, with a good precision, the following equations can be used for positive crystals:

\[
\Delta K_c \approx \pm \sqrt{2(\alpha/c)n_h} \tag{17}
\]

and

\[
\Delta \theta \approx \sqrt{8(c/\alpha)h/n_{\alpha}} \tag{18}
\]

In a principal optical plane of negative crystals, more complicated relations are required for the bandwidth:

\[
\Delta K \approx \sqrt{8(c/\alpha)c} \cdot n_{n_0} n_{n_0} \left( n_2^p \cos^2 \alpha + n_2^e \sin^2 \alpha \right)^{3/4} \tag{19}
\]

and for the deflection angle:

\[
\Delta \theta \approx \sqrt{8(c/\alpha)c} \cdot n_{n_0} n_{n_0} \left( n_2^p \cos^2 \alpha + n_2^e \sin^2 \alpha \right)^{3/4} \cdot n_2^p \cos^2 \alpha + n_2^e \sin^2 \alpha \tag{20}
\]

5. Summary

Elastic anisotropy of crystals manifests itself in sufficient walk-off angles between phase and group velocities of ultrasound. This energy walk-off determines configuration of AOD crystals, and also influences phase matching condition of Bragg acousto-optical diffraction. Optical anisotropy determines the central frequency, and also incidence Bragg angle of light and diffraction angle. The influence of optical anisotropy and rotatory power on the bandwidth of AOD is truly negligible.

The phenomenon of AOD bandwidth shift caused by detuning from tangential phase matching was studied. Analysis showed that the bandwidth shift increases for lower tilt angles of ultrasonic wave vector in the crystal. Moreover, the magnitude of the shift is inversely proportional to the square root of piezotransducer length. Adjustment of AOD with ripples of diffraction efficiency is often used to increase the deflection bandwidth, and frequency shift can be an important factor in design of high-precision AOD-based optical setups.

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References